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SOLUTION BY NATHAN ALTSHILLER, University of Colorado.

Let r be the radical axis of the two given circles γ_1, γ_2, and P the point of intersection of r with the given line, l. The tangents drawn from any point of r, and in particular from P, to all the circles of the coaxial system determined by γ_1 and γ_2, are all of equal length, when measured from P to the respective points of contact. On the other hand, the centers of all the circles coaxial with the two given circles, lie on the line of centers c of these two circles.

The above suggests the following solution of the problem:

From the point P draw a tangent to one of the given circles. Let S denote the point of contact. On l lay off two segments PT and PT' such that $PT = PT' = PS$. The perpendiculars to l erected at T and T' will meet the line of centers c in the centers C and C' of the two circles satisfying the conditions of the problem, the respective radii being the segments CT and $C'T'$.

This construction is applicable whether the given circles are tangent to each other, or cut each other in real, or imaginary points.

The problem has, in general, two real solutions.

An interesting special case arises when the line l is parallel to the radical axis r, and hence perpendicular to the line of centers c. Since the center of the required circle is to be on c, the necessary and sufficient condition for it to be tangent to l is, in the present case, that the circle shall pass through the point of intersection of l with c. We are thus led to the problem:

Given two circles and a point, to draw a circle passing through the given point and coaxial with the two given circles.

This problem may be solved as follows: From an arbitrary point P of the radical axis r draw a tangent PR to one of the given circles, touching the circle at the point R. On the line PQ joining P to the given point Q find the point Q' such that $PQ \cdot PQ' = \overline{PR}^2$, Q and Q' being on the same side of the radical axis. The point Q' belongs to the required circle, which is now readily constructed.

This construction is valid for any point Q in the plane, and whatever the relative position of the two circles with respect to each other may be.

The problem has, in general, one real solution.

Also solved by C. N. SCHMALL, GEO. W. HARTWELL, FRANK IRWIN, HERBERT N. CARLETON, J. W. CLAWSON, and N. P. PANDYA.

469. Proposed by J. ALEXANDER CLARKE, West Philadelphia High School.

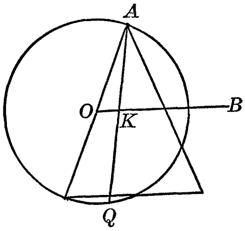
If in an isosceles triangle, a circle is described on one side as diameter, and a line is drawn through the mid-point of the side parallel to the base, the circle and the parallel will intercept on the trisector of the angle at the vertex a segment equal to the radius of the circle. Show how this can be used to trisect any angle.

SOLUTION BY J. W. CLAWSON, Collegeville, Pa.

Let A be the vertex of the isosceles triangle. Bisect either arm at O. Draw the circle, center O, radius OA. Draw a line, OB, from O parallel to the base. Then if a line is supposed drawn to trisect the angle A, cutting the line OB at K and cutting the circle again at Q, we are to prove that $KQ = R$, the radius of the circle.

Now

$$AQ = 2R\cos\frac{A}{3}\,.$$



Also

$$AK = R\frac{\sin(90^\circ - A/2)}{\sin(90^\circ - A/2 + A/3)} = R\frac{\cos\dfrac{A}{2}}{\cos\dfrac{A}{6}}\,.$$

Hence

$$KQ = R\left[2\cos\frac{A}{3} - \cos\frac{A}{2}\sec\frac{A}{6}\right] = R\left[2\cos\frac{A}{3} - 4\cos^2\frac{A}{6} + 3\right]$$

$$= R\left[2\cos\frac{A}{3} - 2\left(1 + \cos\frac{A}{3}\right) + 3\right] = R.$$

Whence, to trisect an angle, construct an isosceles triangle having an arm of the angle for its side and the angle for its vertex. Draw a circle on the side of the isosceles triangle as diameter. Draw from the center of the circle a line parallel to the base of the triangle. Mark off on a straight edge a distance equal to the radius of the circle. Swing and slide the straight edge about until one end of the marked portion of the straight edge coincides with a point of the circle and the other with a point on the line while the straight edge passes through the vertex of the angle.

Also solved by S. A. JOFFE.

469A. Proposed by W. F. FLEMING, Chicago, Ill.

A pole whose length is l stands vertically against a vertical wall. A spider is at each end of the pole. The pole is drawn out from the wall in such a way that its upper end moves down the wall at a uniform rate. At the same time that the pole begins to move, the spiders begin to travel toward each other at rates equal to the rates at which the respective ends move. Determine the equations of the paths of the two spiders, in space.

SOLUTION BY H. S. UHLER, Yale University.

Let (x, y) and (x', y') denote, at any instant, the positions of the spiders which have started from the upper $(0, l)$ and lower $(0, 0)$ ends of the pole respectively. Since the spiders begin to recede from the ends of the pole at the same time that the pole starts to move, and as they crawl along the pole at the same rates as the associated ends of the pole slide along the coördinate axes, the distances which the spiders will have progressed along the pole will, at each instant, be equal to the distances which the corresponding ends of the pole have moved from their initial locations on the axes, quite regardless of whether the upper end of the pole moves uniformly or otherwise. In symbols

$$l - b = +[x^2 + (y - b)^2]^{1/2}, \qquad (1$$

$$a = +[(x' - a)^2 + y'^2]^{1/2}, \qquad (1')$$

where a and b denote the intercepts of the pole-line on the axes of x and y respectively.

Since the spiders are on the pole-line

$$\frac{x}{a} + \frac{y}{b} = 1 \quad (2), \qquad \text{and} \qquad \frac{x'}{a} + \frac{y'}{b} = 1 \quad (2').$$

Since the length of the pole is l, $a^2 + b^2 = l^2$ (3).

The analysis is now reduced to the elimination of a and b from the two sets of equations (1), (2), (3) and (1′), (2′), (3).

Equation (1), when solved for b, gives

$$b = \frac{x^2 + y^2 - l^2}{2(y - l)}. \qquad (4)$$

Substituting b from (4) in (2) we find

$$a = \frac{x(x^2 + y^2 - l^2)}{x^2 - (y - l)^2}. \qquad (5)$$

Substitution of a and b from (5) and (4) in (3) leads to

$$(x^2 + y^2 - l^2)^2 = 4ly(y - l)^2$$

as the equation of the path of the spider which started at the upper end of the pole.

In like manner, we find

$$a = \frac{x'^2 + y'^2}{2x'}, \qquad b = \frac{y'(x'^2 + y'^2)}{y'^2 - x'^2},$$

and

$$(x'^2 + y'^2)^2 = 2lx'(y'^2 - x'^2),$$

which is the equation of the path of the second spider.

The spiders will pass at the point

$$x = \frac{l}{2}(\sqrt{2} - 1), \qquad y = \frac{l}{2},$$